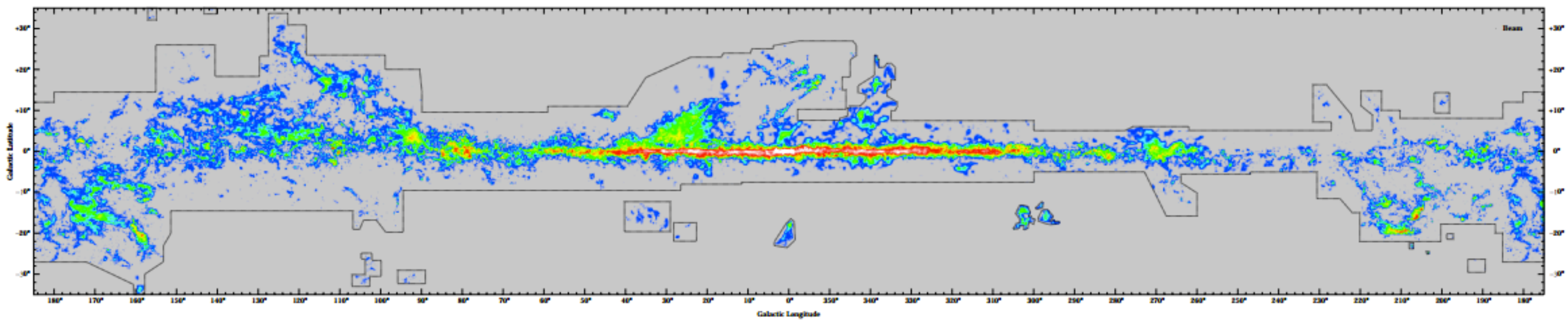
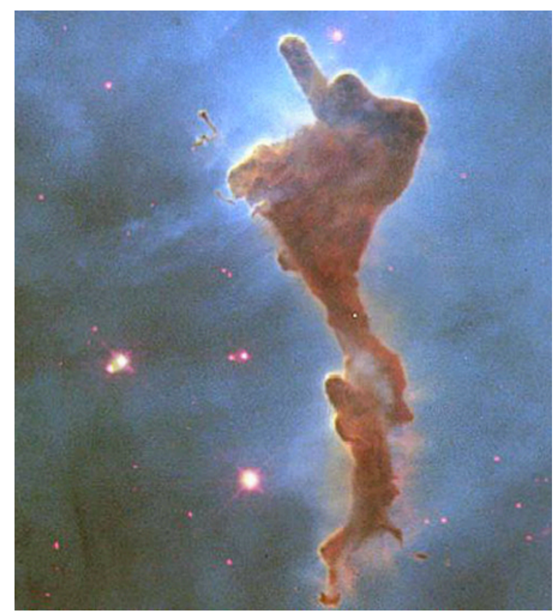
Molecular Clouds



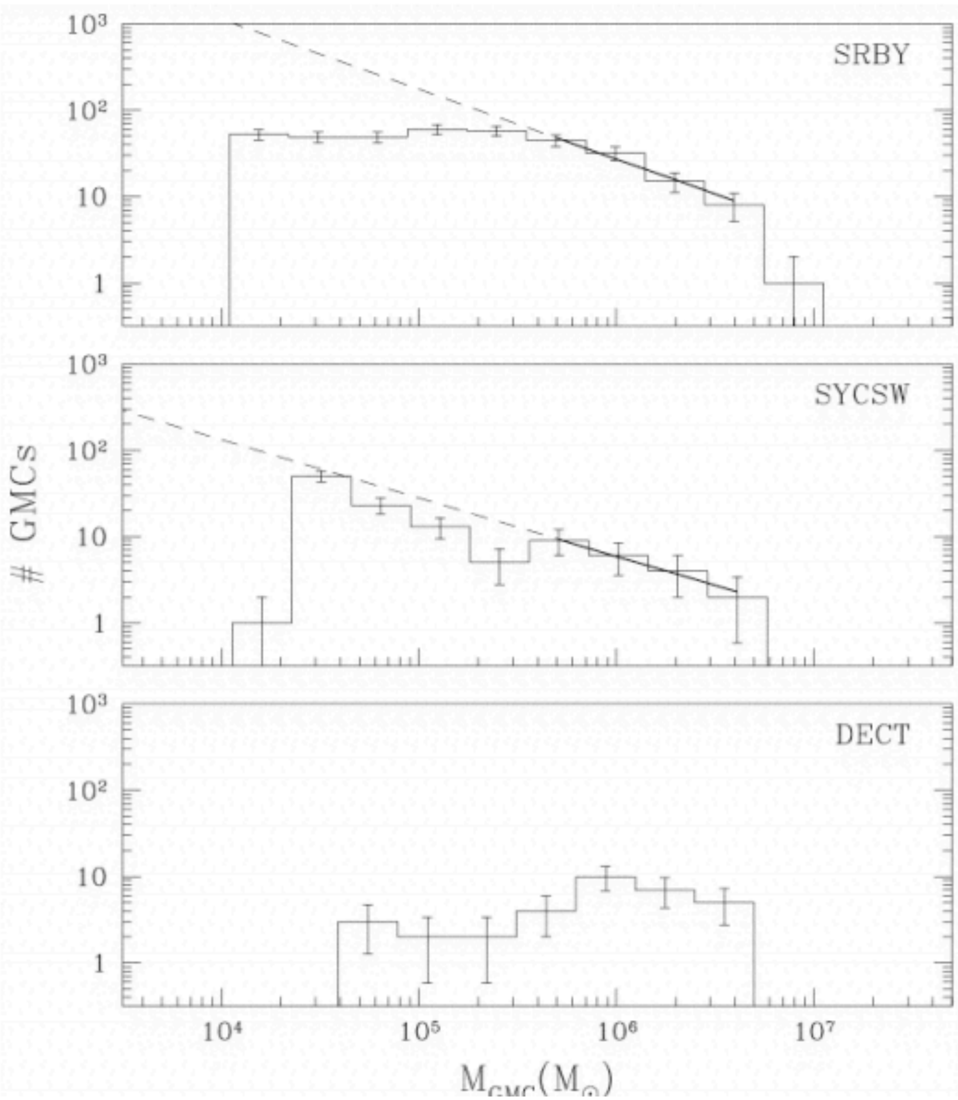
The Milky Way in Molecular clouds
http://www.cfa.harvard.edu/mmw/MilkyWayinMolClouds.html



Molecular Clouds Properties

- Sizes: 10-100 pc
- Masses: 10 to 10^6 $M_\odot$. Most of the molecular gas mass in galaxies is found in the more massive clouds.
- Average Density: 100 cm^{-3}
- Composition: H_2, He, dust (1% mass), CO (10^{-4} by number), and many other molecules with low abundances
- Lifetimes: < 10 Myr

Cloud Mass Function



$dN/dlog(m) = kM^{-0.81}$

$M_{upper} = 5.8 \times 10^6$ Msun

$dN/dlog(m) = kM^{-0.67}$

$M_{upper} = 5.8 \times 10^6$ Msun

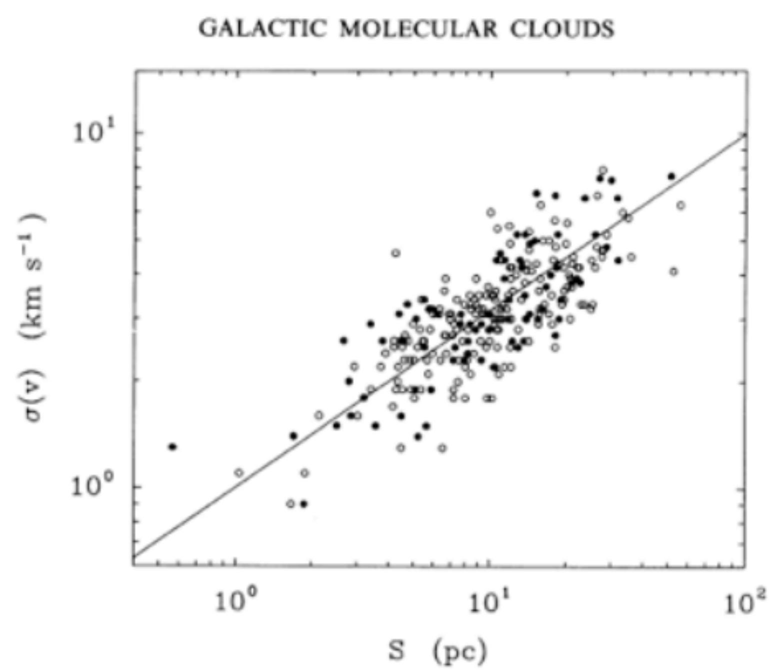
<u>Mass is weighted toward the most massive clouds</u>

Williams & McKee 1997 ApJ 476, 166.

Larson's Law

- In 1981, Richard Larson wrote a paper titled: "Turbulence and Star Formation in Molecular Clouds" (MNRAS 1984, 809)

- Used tabulation of molecular cloud properties to discover to empirical laws based on the cloud length L (max. projected length
- Size-line-width relationship: σ_v (km s^{-1}) = 1.10 $L^{0.38}$ (pc)
  - more recent values give σ_v = const $L^{0.5}$



Solomon 1987 ApJ 319, 730

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Virial Theorem Analysis

Fundamental theorem which allow us to assess generally the balance of forces within *any* structure in hydrostatic equilibrium. In the particular case of clumps within complexes, it will allow us to understand if the magnitude of the observed velocity dispersion is consistent with the internal gravitational field.

Statement of the Theorem

Equation of motion for an inviscid fluid:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla P - \rho \nabla \phi_g + \frac{1}{c} \mathbf{j} \times \mathbf{B}$$

Full or convective time derivative of the fluid velocity **u**

Magnetic force per unit volume acting on a current density **j**.

Using Ampère's Law and the vector identity for triple cross products:

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla P - \rho \nabla \phi_g + \frac{1}{4\pi}(\mathbf{B} \bullet \nabla)\mathbf{B} - \frac{1}{8\pi}\nabla |\mathbf{B}|^2$$

Tension associated with curved magnetic field lines

Gradient of a scalar *magnetic pressure* of magnitude $|\mathbf{B}|^2/8\pi$

The existence of tension requires bending of the field lines, while pressure arises from the crowding of the lines, whether they be curved or straight.

The above equation governs the *local* behaviour of the fluid. To derive a relation between *global* properties of the gaseous body, let's form the scalar product of the hydrostatic equation with the position vector **r** and integrate over volume.

Using the mass continuity and Poisson's equations, neglecting the external pressure (valid for the strongly self-gravitating giant complexes, but not for HI or diffuse clouds), repeated integration by parts leads to the virial theorem:

$$\frac{1}{2}\frac{\partial^2 I}{\partial t^2} = 2\mathcal{T} + 2\mathcal{U} + \mathcal{W} + \mathcal{M}$$

Moment of inertia ➔	$I \equiv \int \rho \,\lvert \mathbf{r} \rvert^2 d^3\mathbf{x}$
Total kinetic energy ➔	$\mathcal{T} \equiv \frac{1}{2}\int \rho \,\lvert \mathbf{u} \rvert^2 d^3\mathbf{x}$
Energy in random, thermal motion ➔	$\mathcal{U} \equiv \frac{3}{2}\int n k_B T d^3\mathbf{x} = \frac{3}{2}\int P d^3\mathbf{x}$
Gravitational potential energy ➔	$\mathcal{W} \equiv \frac{1}{2}\int \rho \phi_g \, d^3\mathbf{x}$
Magnetic energy ➔	$\mathcal{M} \equiv \frac{1}{8\pi}\int \lvert \mathbf{B} \rvert^2 d^3\mathbf{x}$

The issue here is which of the above terms can balance the gravitational binding energy $\mathcal{W}$. If *none*, then a typical cloud would be in a state of gravitational collapse.

Free-fall time

Assuming that a cloud with radius R and mass M is in gravitational collapse:

$$\frac{1}{2}\frac{\partial^2 I}{\partial t^2} \approx -\frac{GM^2}{R}$$

Approximating *I* as MR^2, then R in the collapsing cloud shrinks by ~ a factor of 2 over a characteristic *free-fall time* t_{ff}:

$$t_{ff} \approx \left(\frac{R^3}{GM}\right)^{1/2} = 7\times 10^6\,\mathrm{yr}\left(\frac{M}{10^5 M_{sun}}\right)^{-1/2}\left(\frac{R}{25\ pc}\right)^{3/2}$$

Since $M/R^3 \approx \rho$, the time scale can also be written as $(G\,\rho)^{-1/2}$. Conventionally, t_{ff} is defined by the time for a homogeneous sphere with zero internal pressure to collapse to a point:

$$t_{ff} \equiv \left(\frac{3\pi}{32G\rho}\right)^{-1/2}$$

Support of Giant Complexes

If the complexes are in approximate force balance over their lifetimes, we can consider the form of the virial theorem appropriate for longterm stability:

$$2\mathcal{T} + 2U + \mathcal{W} + \mathcal{M} = 0$$

For a cloud is such "virial equilibrium", what is balancing $\mathcal{W}$?

1. Thermal energy ??

$$\frac{U}{|\mathcal{W}|} \approx \frac{M\mathcal{R}T}{\mu}\left(\frac{GM^2}{R}\right)^{-1} = 3\times10^{-3}\left(\frac{M}{10^5 M_{sun}}\right)^{-1}\left(\frac{R}{25\ pc}\right)\left(\frac{T}{15\ K}\right)$$

↓

NO

2. Magnetic energy ??

$$\frac{\mathcal{M}}{|\mathcal{W}|} \equiv \frac{|\mathbf{B}|^2 R^3}{6\pi}\left(\frac{GM^2}{R}\right)^{-1} = 0.3\left(\frac{B}{20\mu G}\right)^2\left(\frac{R}{25\ pc}\right)^4\left(\frac{M}{10^5\ M_{sun}}\right)^{-2}$$

Magnetic fields are important!

$$\rho\frac{D\mathbf{u}}{Dt} = -\nabla P - \rho\nabla\phi_g + \frac{1}{c}\mathbf{j}\times\mathbf{B}$$

➔ The force associated with the magnetic field acts on a fluid element in a direction orthogonal to **B**. Thus, any self-gravitating cloud can slide freely along field lines (but no flattening observed!).

The observed scatter in the alignment of the observed **E** vectors indicates the presence of a *random* component of **B**, coexisting with the smooth background. The field distortion arises, at least in part, from magneteto-hydrodynamic (MHD) waves, which may provide the isotropic support preventing the cloud from flattening.

3. Kinetic energy ??

The bulk velocity within giant clouds stems mostly from the random motions of their clumps (ΔV is the mean value of the random motion of clumps in GMCs):

$$\frac{T}{|W|} \approx \frac{1}{2} M\,\Delta V^2 \left(\frac{GM^2}{R}\right)^{-1} = 0.5 \left(\frac{\Delta V}{4 km/s}\right)^2 \left(\frac{R}{25\ pc}\right) \left(\frac{M}{10^5\ M_{sun}}\right)^{-1}$$

ΔV is close to the *virial velocity*, defined as:

$$V_{vir} \equiv \left(\frac{GM}{R}\right)^{1/2}$$

$$\boxed{t_{ff} \approx \left(\frac{R^3}{GM}\right)^{1/2}}$$

➔ V_{vir} is the velocity of a parcel of gas that traverses the cloud over the t_{ff}, i.e. it is the typical speed attained by matter under the influence of the cloud's internal gravitational field.

Reading Assignment

- Read Chapter 3 and Chapter 9 in The Formation of Stars (Stahler and Palla)